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AFFINE EQUIVALENCE AND ITS APPLICATION TO TIGHTENING THRESHOLD IMPLEMENTATIONS

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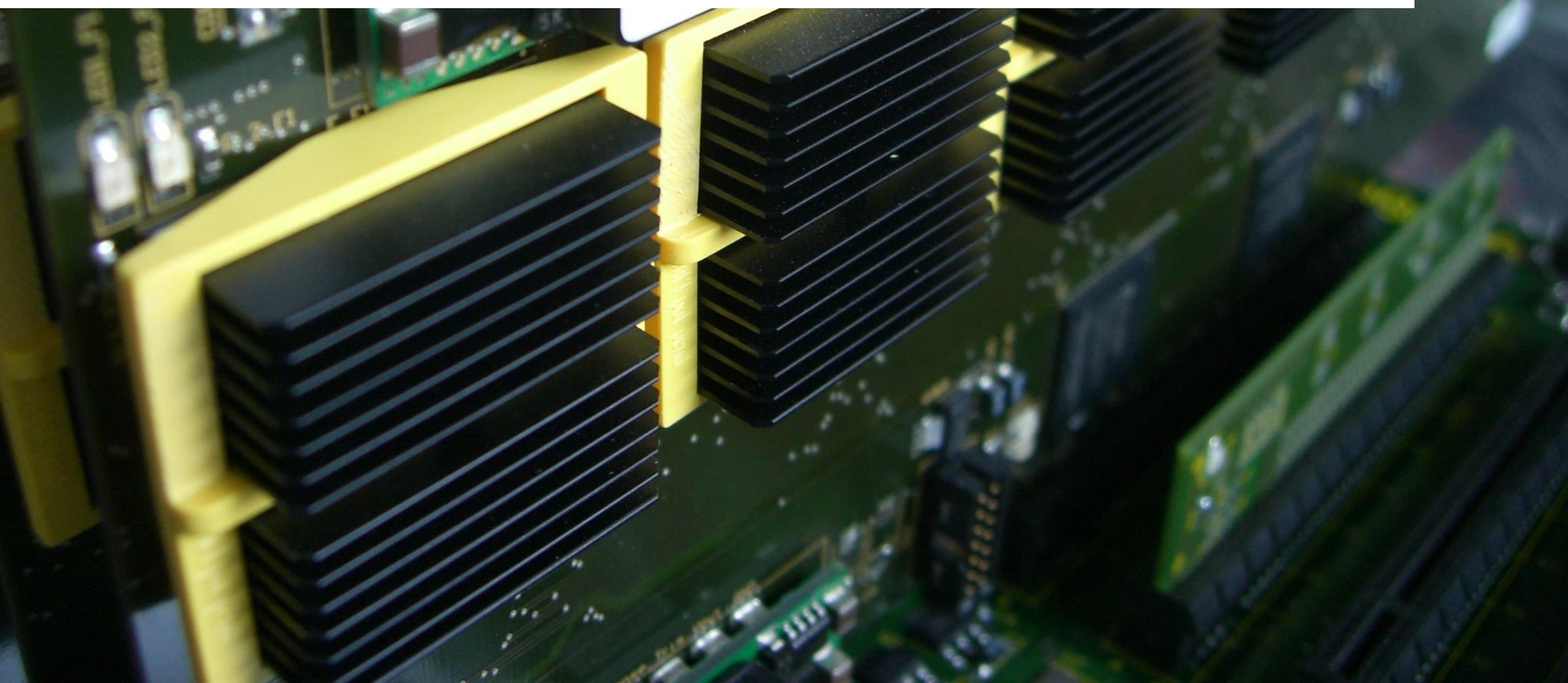
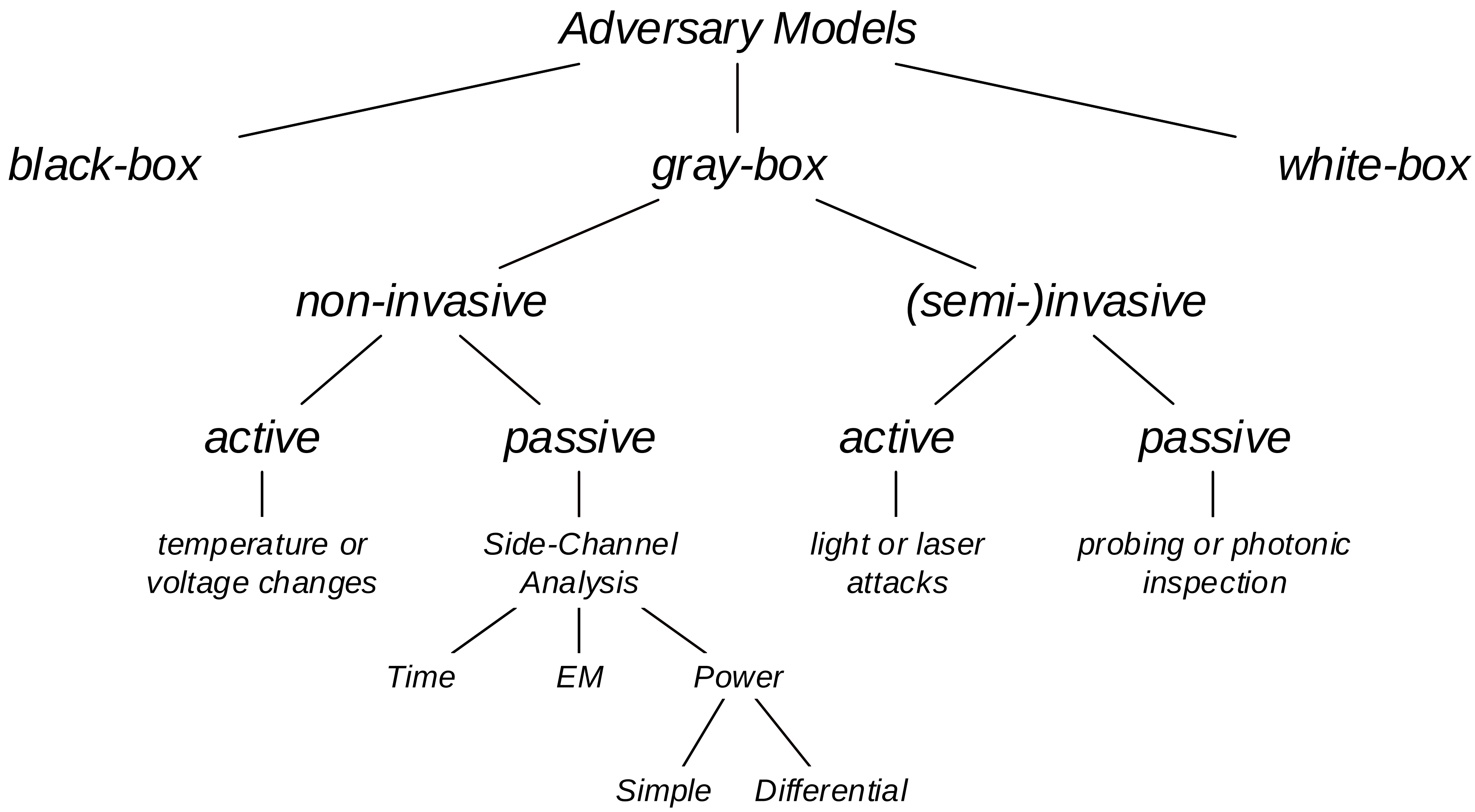


WHAT ARE THRESHOLD IMPLEMENTATIONS?

***Threshold Implementations* are a countermeasure against Side-Channel Analysis such as Differential Power Attack.**

SIDE-CHANNEL ANALYSIS (SCA)



DIFFERENTIAL POWER ANALYSIS (DPA)

General: Measure multiple power traces of an encryption with *same key* but *different plaintexts*.

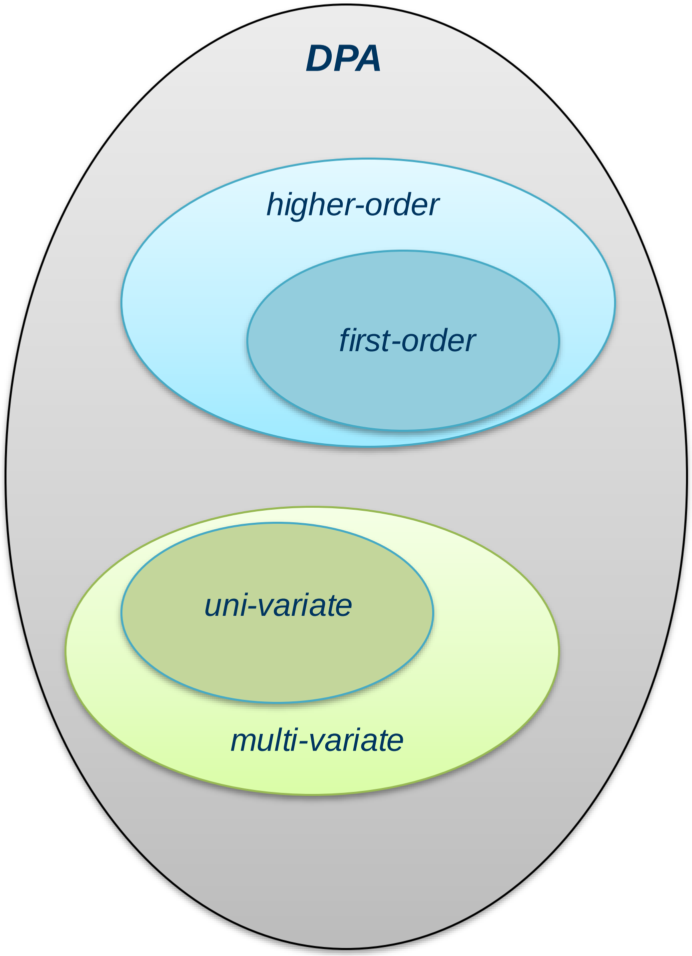
Idea: Each signal transition will consume a different amount of power.

- 0 → 0: *low*
- 0 → 1: *high*
- 1 → 0: *high*
- 1 → 1: *low*

The leakage of an encryption $E_k(m)$ will create a unique fingerprint in the power consumption.

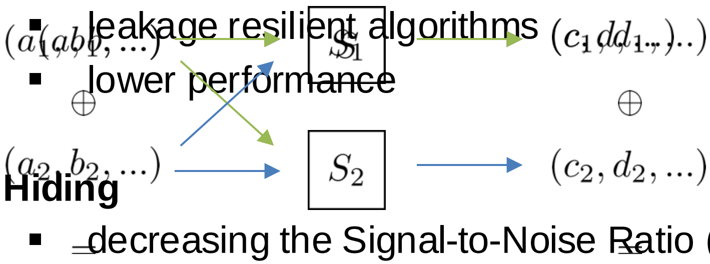
Statistical analysis will help to reveal the secret encryption key k.

Analysis is simplified using divide-and-conquer strategies (e.g. only observing S-box computation)



COUNTERMEASURES AGAINST DIFFERENTIAL POWER ANALYSIS

1. **Limitation of the key invocation**
   - key distribution is a challenge
   - leakage resilient algorithms
   - lower performance



2. **Hiding**
   - decreasing the Signal-to-Noise Ratio (SNR)
   - decrease signal (e.g. power equalization, logic styles)
   - increase noise (e.g. shuffling, dummy executions)

3. **Masking**
   - randomizing the leakage
   - secret sharing approach
   - multi-party computations

unshared	shared	HW	mean	var
0	$(0,0)$ $(1,1)$	0 2	1	2
1	$(0,1)$ $(1,0)$	1 1	1	0

✓ first-order DPA security

X second-order DPA security

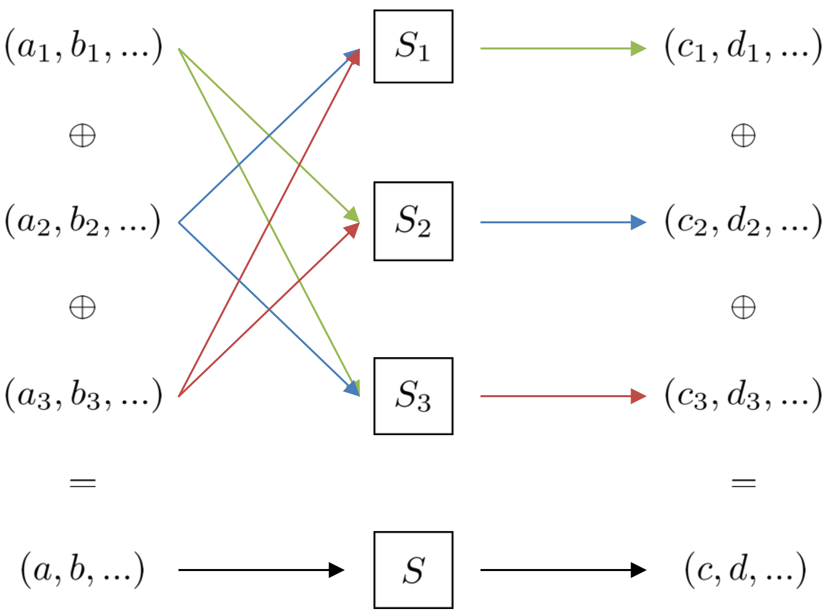
THRESHOLD IMPLEMENTATION

Threshold Implementation:

- efficient *countermeasure* against (*first-order*) Side-Channel Analysis
- introduced in 2006 by Nikova et al.
- provides provable security even in a glitch circuit

Concept and properties:

- uniform masking
- non-completeness
- correctness
- uniform sharing of function outputs (each set of output pairs occurs with same probability)

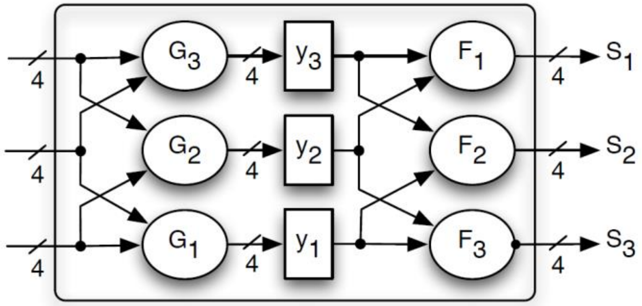


Note: *The number of input and output shares depends on the function S.*

WHAT IS THE STORY OF THIS WORK?

- **Side-Channel Analysis (SCA):** attacks exploit information leakage of cryptographic devices
- **Threshold Implementation (TI):** countermeasure based on Boolean masking and multi-party computation

Problem: TI counteracts *first-order attacks*, but is vulnerable to *higher-order attacks* (using higher-order statistical moments).



Different approaches to encounter this problem:

- **Higher-order Threshold Implementations (HO-TI)**
  - *might be* restricted to univariate settings
  - area overhead *might be* problematic
  - finding uniform representations *might be* challenging
- **Stay with 1st-order secure TI and make *higher-order attacks* harder**
  - increase the noise
  - reduce the signal

Our contribution: Increase the noise by introducing structured randomness into a 1st-order secure TI.

NOISE ADDITION

Started a **case study** on PRESENT cipher:

- particularly investigated the PRESENT S-box $y = S(x)$
- S-box has algebraic degree of 3, at minimum 4 shares
- alternatively, S-box can be decomposed into quadratic functions
- thanks to classification in

Bilgin, Nikova, Nikov, Rijmen, Tokareva, Vitkup: Threshold implementations of small S-boxes. Cryptography and Communications 7(1): 3-33 (2015)

we know that PRESENT S-box $S : A' \circ C_{266}^4 \circ A$ can be decomposed in 7 different ways

$(\mathcal{Q}_{12} \circ \mathcal{Q}_{12})$, $(\mathcal{Q}_{293} \circ \mathcal{Q}_{300})$, $(\mathcal{Q}_{294} \circ \mathcal{Q}_{299})$, $(\mathcal{Q}_{299} \circ \mathcal{Q}_{294})$, $(\mathcal{Q}_{299} \circ \mathcal{Q}_{299})$,
$(\mathcal{Q}_{300} \circ \mathcal{Q}_{293})$, $(\mathcal{Q}_{300} \circ \mathcal{Q}_{300})$

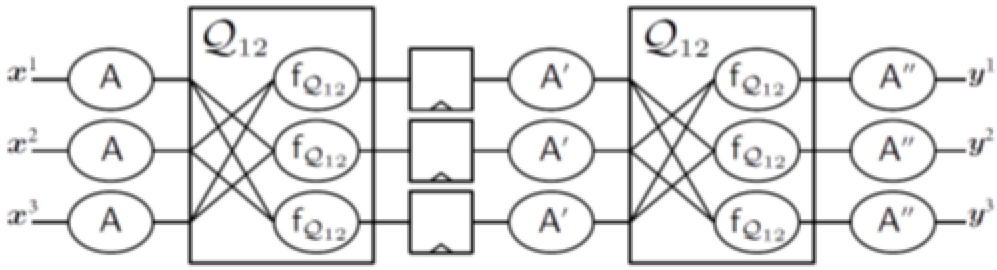
- it means, e.g.: $S : A'' \circ \mathcal{Q}_{12} \circ A' \circ \mathcal{Q}_{12} \circ A$ with three affine functions (A, A', A'')

Idea: Randomly change affine functions on-the-fly to introduce structured (random) noise.

SOME NOTES ON AFFINE FUNCTIONS

Question: How can we implement, e.g. $S : A'' \circ \mathcal{Q}_{12} \circ A' \circ \mathcal{Q}_{12} \circ A$ with random affine functions?

Solution: Uniform TI of $\mathcal{Q}_{12}$ is easily made by *direct sharing*



Question: How many of such 3-tuple affine functions exist (depending on the decomposition)?

Decomposition	# of (A, A', A'')
$\mathcal{Q}_{12} \circ \mathcal{Q}_{12}$	147 456
$\mathcal{Q}_{294} \circ \mathcal{Q}_{299}$	229 376
$\mathcal{Q}_{299} \circ \mathcal{Q}_{294}$	229 376
$\mathcal{Q}_{299} \circ \mathcal{Q}_{299}$	200 704

Note: We exclude those decompositions with $\mathcal{Q}_{300}$, as its uniform TI needs (at least) two stages.

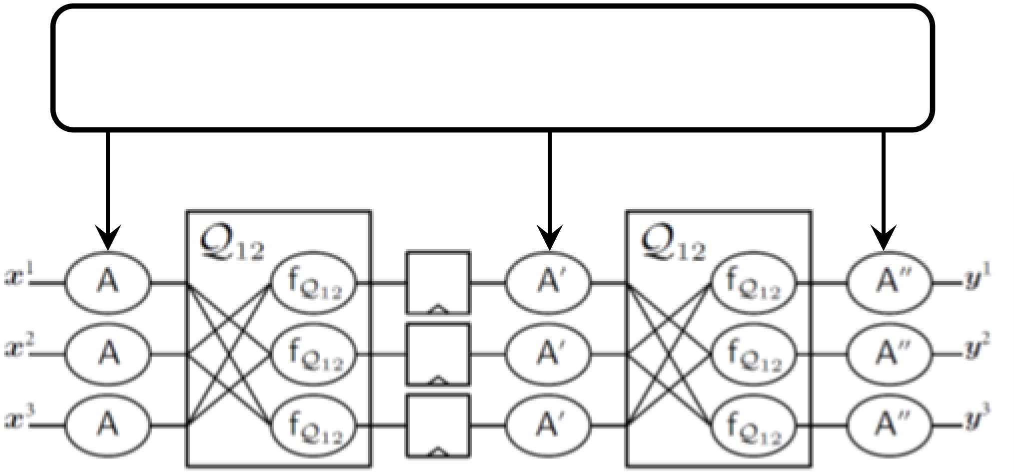
CHANGING THE AFFINE FUNCTIONS ON-THE-FLY

Implementation platform: Spartan-6 FPGA of SAKURA-G side-channel board.

Focused on decomposition $S: A'' \circ \mathcal{Q}_{12} \circ A' \circ \mathcal{Q}_{12} \circ A$ with 147 456 different 3-tuple affine functions.

Hope: If we change the affine functions dynamically, this will introduce random noise to our design and make *second-order attacks* harder (but it should not affect the *third-order vulnerability)*.

Challenge: How can we implement a circuit that allows us to select a random 3-tuple affine function of the set of all possible (e.g. 147 456) affine functions?



OPTION 1: SAVE ALL AFFINE TRANSFORMATIONS

Naïve approach that precomputes and stores all affine transformations on the target device.

Single affine transformation: 4×4 binary matrix and 4-bit constant (20 bit)

All affine transformations: $3 \times 147\,456 \times 20$ bit $= 8640$ kbit

Problem: Spartan-6 LX75 FPGA (*XC6SLX75*) has only 3096 kbit dedicated block memory (BRAM).

Solution: Precompute and store only a fraction of all possible affine triples. For example, 16384 affine triples would occupy 60 BRAMs.

Disadvantages:

- approach is extreme costly in terms of area (memory) requirements
- only covers a fraction of all possible affine functions which may reduce the security

OPTION 2: SEARCH AFFINE TRANSFORMATIONS ON-THE-FLY

This approach just implements the searching algorithm to precompute the affine triples in hardware.

Advantages:

- pretty efficient in terms of area (memory) overhead
- covers all possible (e.g. $147\,456$) options to select an affine triple

Disadvantages:

- affine triples are not found with a constant rate (i.e. algorithm is not time-invariant)
- several affine triples are found sequentially and for a long time no new affine triple may be found
- it may happen that multiple encryptions are performed with a fixed set of affine functions (contradiction with our goal)

OPTION 3: GENERATE AFFINE TRANSFORMATIONS ON-THE-FLY

This approach uses some interesting observations to reduce the number of affine triples to be stored.

Observations for $S : A'' \circ \mathcal{Q}_{12} \circ A' \circ \mathcal{Q}_{12} \circ A$ **:**

- only 384 different input affine functions A
- only 384 different output affine functions A''
- $384 \times 384 = 147456$ different combinations of A and A''
- set of 384 affine functions is made of 48 linear functions combined with 8 different constants

Idea: Store only input and output affine functions and compute middle the affine function on-the-fly.

Approach:

- compute the middle affine using the inverses of A and A''
- $48 \times 2 \times 16$ bit storage for linear and inverse of A (same for A'')
- $2 \times 48 \times 2 \times 16 = 3$ kbit in total, which fits into a single BRAM
- Some extra logic to compute middle affine

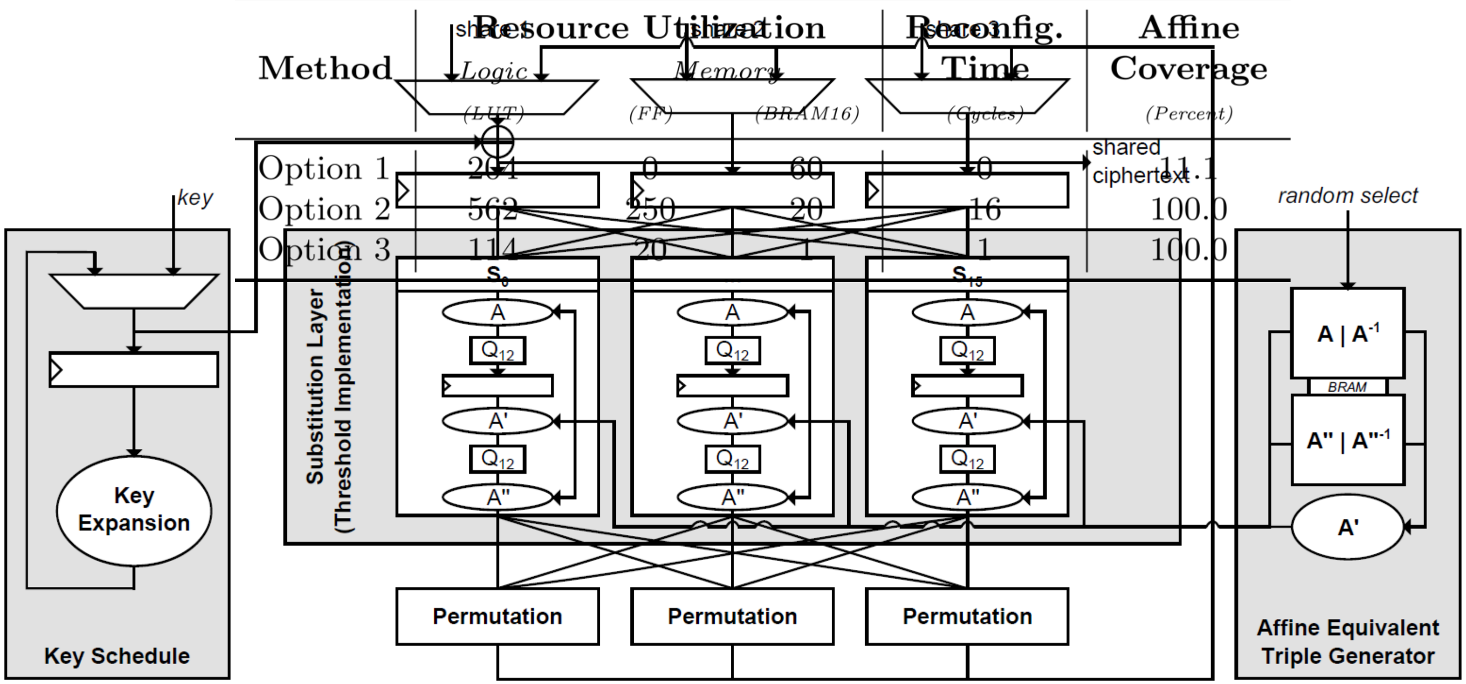
IMPLEMENTATION OF THE CASE STUDY

We implemented PRESENT-128 following a round-based fashion.

- pipeline with two stages, due to the middle register in the decomposed S-box
- 33 clock cycles latency with two full encryptions

We also provide a comparison between the options to realize the random affine selection:

Method	Resource Utilization			Reconfig. Time	Affine Coverage
	Logic (*LUT*)	*Memory* (*FF*)	*Memory* (*BRAM16*)	(*Cycles*)	(*Percent*)
Option 1	201	0	60	0	11.1
Option 2	562	250	20	16	100.0
Option 3	114	20	1	1	100.0

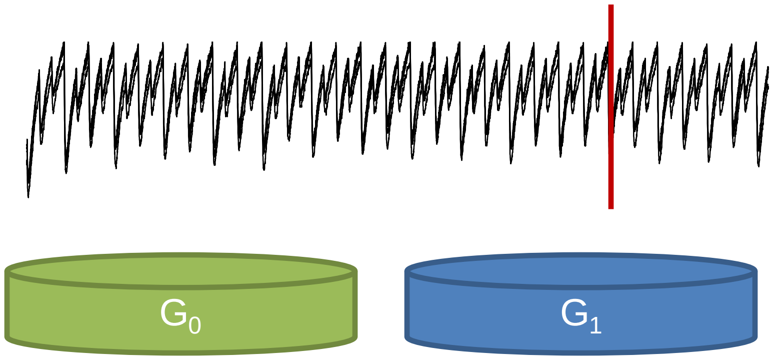


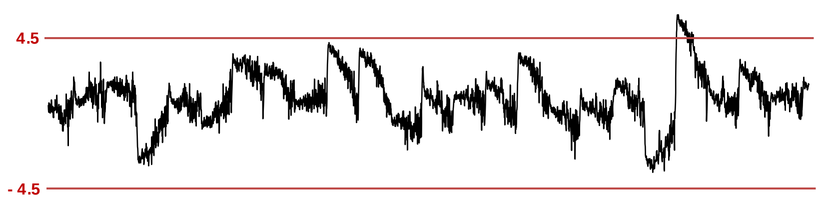
EVALUATION BY WELCH'S *t*-TEST

- measure power traces with digital oscilloscope
- determine distinguisher, e.g.:
  - fix vs. random plaintext (*non-specific t*-test)
- group traces depending on distinguisher
- compute *sample mean* for each point in time
- compute *sample variance* for each point in time
- determine *t*-statistic for each point in time:

$$t = \frac{\mu(T \in G_1) - \mu(T \in G_0)}{\sqrt{\frac{\delta^2(T \in G_1)}{|G_1|} + \frac{\delta^2(T \in G_0)}{|G_0|}}}$$

where μ denotes the *sample mean* and δ denotes the *sample variance*.



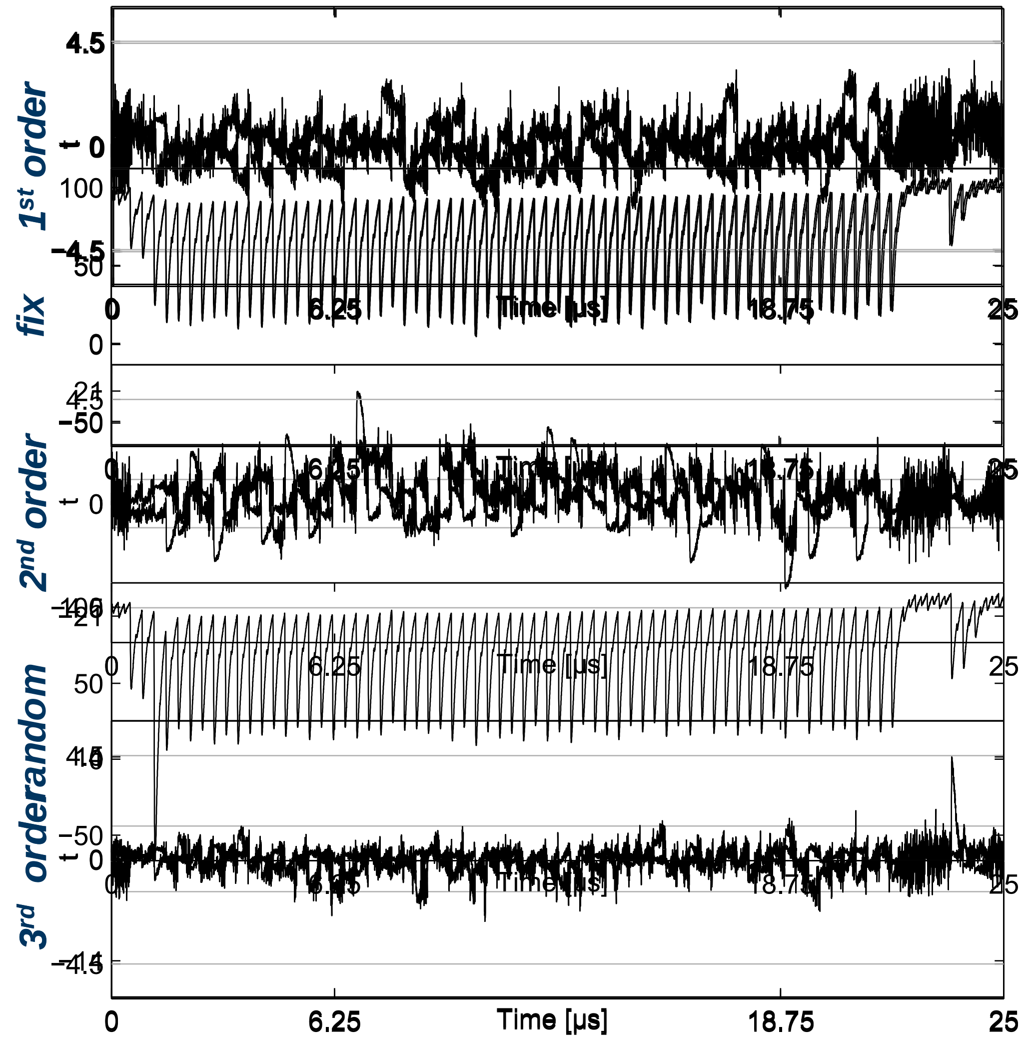


Fail/Pass Criteria: If there is any point in time for which the t-statistic exceeds a threshold of $\pm$ 4.5 the device under test fails.

More info: "Leakage Assessment Methodology - a clear roadmap for side-channel evaluations", Cryptology ePrint Archive, Report 2015/207

RESULTS

- Sample trace for:
  - fix affine triples
  - random affine triples
- first-order, second-order and third-order *non-specific t*-test:
  - fixed affine triples (50 million traces)
  - random affine triples (200 million traces)
- as expected no first-order leakage detected (TI)



CONCLUSION:

Changing the affine triples randomly could avoid detectable second- and third-order leakage.

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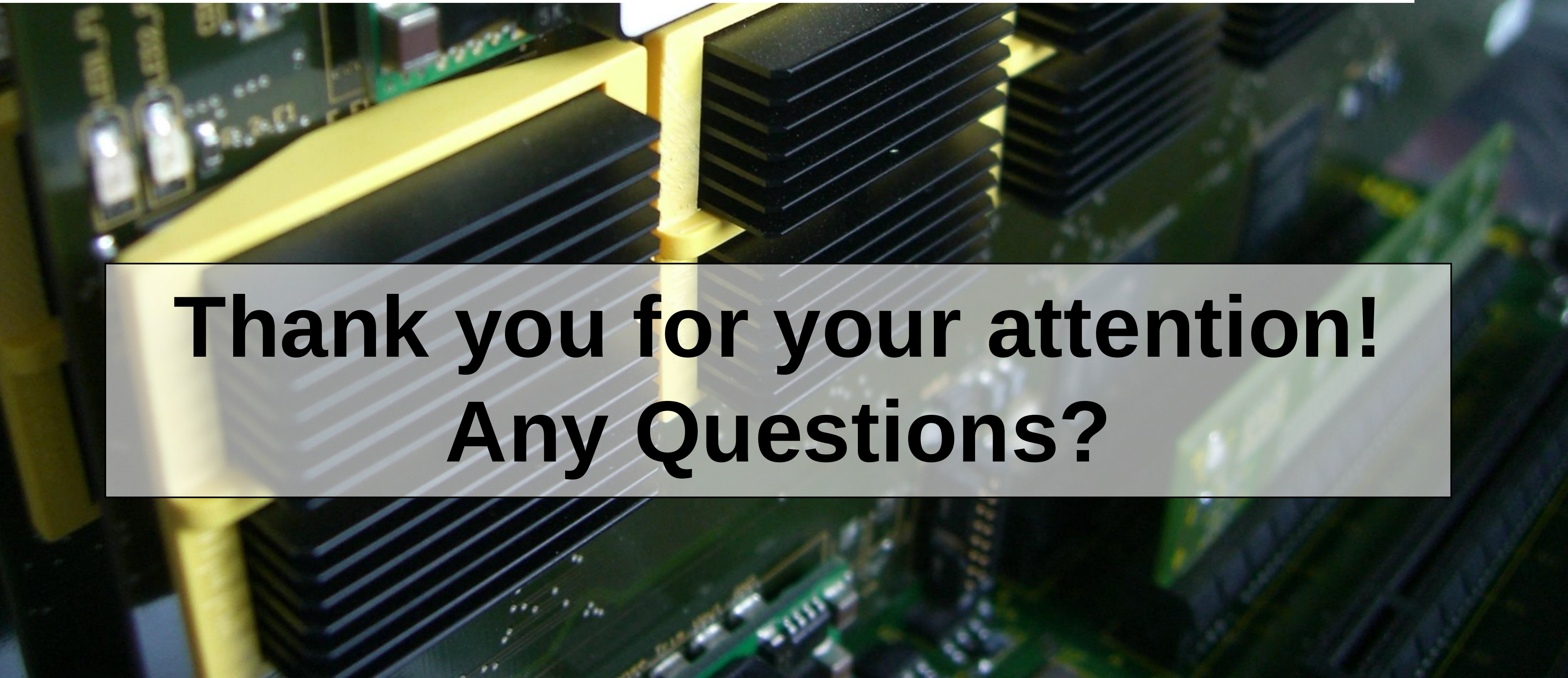

Thank you for your attention!

Any Questions?